Centre for Globalization Research
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CGR Working Paper 65

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An individual-based approach to the measurement of multiple-period mobility for nominal and ordinal variables*

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June 29, 2016

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*We are very grateful to Andrew Clark, Marek Kosny and seminar participants at the OECD-Universities Joint Conference, Paris, 3-5 July 2013 and the Fifth ECINEQ Meeting, Bari, 22-24 July 2013, for their very helpful comments and suggestions.

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1 Introduction

The study of fluctuations (and stability) in socioeconomic outcomes has long been of interest in the social sciences, often under the name of intra-generational economic mobility. Fields and Ok (1996) discuss several concepts of this form of mobility. Most of them apply to two-period analyses, in which several topics of interest include decomposition into structural and exchange components (e.g. Ruiz-Castillo, 2004, van Kerm, 2004), as well as the "pro-poor" nature of a growth experience (e.g. see review by Deutsch and Silber, 2011). Meanwhile, the literature on mobility over several periods has been concerned mainly with the notion of mobility as an equalizer of lifetime incomes (e.g. Shorrocks, 1978a, Maasoumi and Zandvakili, 1986, Tsui, 2009, Fields, 2010). This literature deals with continuous variables.

The setting of two-period mobility with ordinal variables has also been developed extensively, especially using transition matrices (e.g. Prais, 1955, Shorrocks, 1978b, Sommers and Conlisk, 1979, Bartholomew, 1982, Dardanoni, 1995, Van de Gaer, Schokkaert, and Martinez, 2001, Parker and Rougier, 2001). In contrast, mobility with nominal and ordinal variables in a multiple-period framework has not been explored yet. However some interesting questions are worth tackling in this context. For instance, how stable over time are people's expressions of life satisfaction (measured by ordinal indicators) or lifetime voting patterns (measured by nominal variables)? To what extent are these affected by life shocks, or conditioned by more stable socioeconomic and demographic characteristics? With datasets like the British Household Panel Survey answering these questions is now possible.

This paper discusses meanings of intra-generational mobility when variables take values that are either unordered or ordered categories. We first propose a concept of mobility as diversity, unpredictability and/or instability in people's status over the accounting period. This notion of mobility is relevant and applicable to both nominal and ordinal variables. For this notion, we propose sensible benchmarks of maximum and minimum mobility, along with mobility-inducing transformations and related desirable properties. Then we axiomatically characterize indices of individual mobility, and social mobility, for this particular meaning. It turns out that the individual mobility indices are consistent with a mobility quasi-ordering that is, essentially, a Lorenz quasi-ordering mapping from discrete probability distributions.

Our concept and measures of mobility as diversity, instability or unpredictability is similar to the notion of mobility as unpredictability put forward by Parker and Rougier (2001). The main difference between the two concepts (and related indices) is that ours focuses on observing individuals over several periods and our measures evaluate *individual* mobility experiences, with the option of aggregating these individual mobility measures into a social measure at a latter stage. By contrast, the concept of Parker and Rougier (2001) is based on two-period transition matrices and skips the individual evaluation stage.

As a second contribution, we also propose a concept of *ordinal* individual mobility as average distance traveled across categories between adjacent periods. This notion is only relevant for ordinal variables measuring people's status (e.g. subjective wellbeing). Likewise we propose corresponding benchmarks of maximum and minimum ordinal mobility, together with ordinal-mobility-inducing transformations and related desirable properties. The we axiomatically characterize indices of individual ordinal mobility, from which social indices can also be constructed, reflecting this particular meaning.

Our concept and measures of ordinal mobility is also similar to the notion of mobility as movement described by Van de Gaer et al. (2001). In particular, it is similar in how it is expressed using a transition-matrix index by Bartholomew (1982), which weights the absolute value of the gaps between the departure and destination categories (measured with assigned natural numbers) by the corresponding transition probabilities, thereby being sensitive to the average distance traveled between categories from one period to the next. Again, the main difference between our concept and those similar to it, is that we focus on individual observations over several periods, with the option of aggregating their individual mobility measures into a social measure later on. In contrast, the concept of mobility as movement as captured by the Bartholomew index among others, is based on two-period transition matrices and implicitly skips the individual evaluation stage.

We apply these concepts and indices in order to measure the extent of mobility as instability, as well mobility as movement, in a multiple-period setting, using responses to subjective wellbeing questions in the United Kingdom using the British Household Panel Survey. In particular, we study socioeconomic covariates of individual mobility indices, with a special concern for potential explanatory factors behind both low mobility and high mobility values. Are these extreme values explainable by socioeconomic variables, or do they rather reflect survey response errors? We find that individuals who experience low mobility in their life satisfaction are educated and well-off, most often female, married, in good health, and with an insignificant incidence of any unemployment spells. Meanwhile, individuals with high mobility in life satisfaction, on the other hand, are almost always male, single and young, with low levels of education and income, and often ill.

The paper is organized as follows. In Section 2 we set the framework for conceptualising and measuring multiple-period mobility as instability or unpredictability. Section 3 provides our proposal for conceptualising and measuring multiple-mobility with specifically ordinal variables. In Section 5 we illustrate the use of these measures with an empirical study of life satisfaction responses from the British Household Panel Survey. Section 6 concludes.

2 Multiple-period mobility as unpredictability

2.1 Preliminaries and notation

Let x_{nt} be the value of a categorical (ordered or unordered) variable x for individual n in period t, such that $x_{nt} \in [1, S] \subset \mathbb{N}_{+}$. The variable is observed for N individuals across T time periods. The unobserved individual probability that $x_{nt} = i$ in any time period is: $p_n(i) \equiv \Pr[x_{nt} = i]$. In practice, the probability that $x_{nt} = i$ is estimated according to the following formula:

$$\widehat{p_n(i)} \equiv \frac{1}{T}\sum_{t=1}^{T}\mathbb{I}(x_{nt} = i) \tag{1}$$

We also define the discrete probability distribution of x for individual n, an S-dimensional vector, as: $P_n := [p_n(1), p_n(2), ..., p_n(S)]$. An analogue definition applies to the estimated probability distribution, $\widehat{P_n}$.

2.2 Mobility benchmarks and transformations

When we observe an unordered categorical variable for each individual during several time periods we can ask: how stable is the individual's experience according to the variable? Does the individual always report the same value? Or, on the other extreme, is the individual likely to report any possible value from one period to the next in such a way that in each period the immediate future value of the variable is unpredictable? We can also pose these questions on instability and/or unpredictability of responses to ordinal variables.

For example, figure 1 shows the response patterns of individuals "A" and "B" to a job occupation question with six answer categories, over five years. Individual "A" always answers category "2", whereas individual "B" answers different categories in every year. Clearly, the pattern of "A" is stable, and highly predictable. By contrast "B" exhibits a highly unstable pattern, difficult to predict. Hence, arguably, an interesting question is to what extent can these different patterns be explained by fluctuations (or lack thereof) in the events of individuals' lives. When surveying subjective wellbeing we may also be interested in knowing whether the extreme cases, in particular, (e.g. those of "A" or "B") reflect appraisals of actual life experiences or bad respondent behaviour.

In this context, we introduce the following two extreme situations, or benchmarks, of mobility understood as diversity (of outcomes), unpredictability or instability:

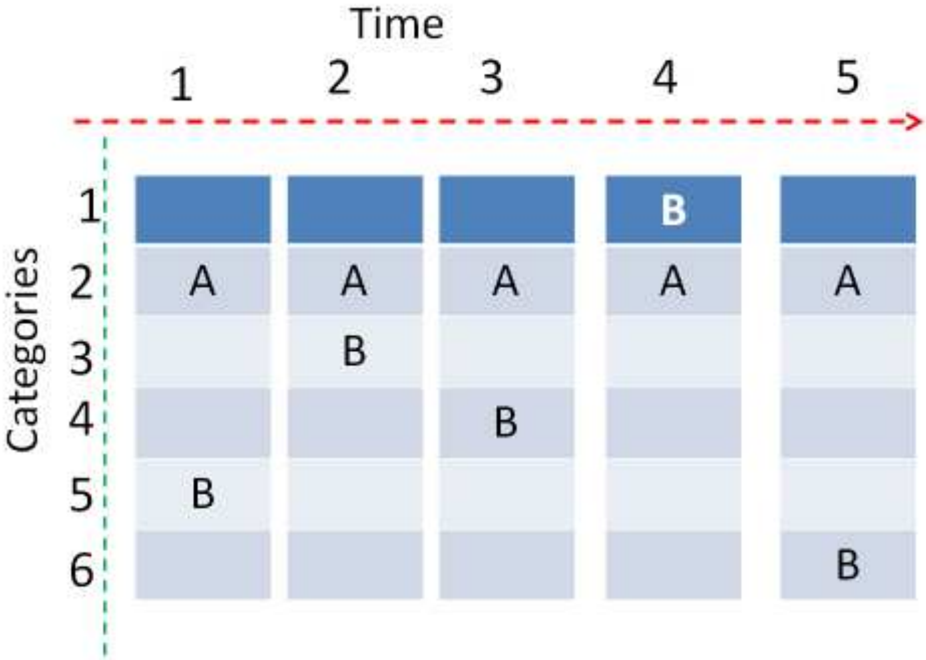
Definition 1. *Minimum mobility: An individual experiences minimum mobility if* $\exists i \in [1, S] \mid p_n(i) = 1$.

According to definition 1, minimum mobility occurs when an individual always reports the same value of x.

Definition 2. *Maximum mobility: An individual exhibits maximum mobility if* $p_n(j) = \frac{1}{S}\ \forall j \in [1, S]$.

According to definition 2, maximum mobility (understood as diversity/instability/unpredictability) occurs when the individual probability distribution of x is uniform. Note that these two extremes would conform with the benchmarks of best and worst predictability of outcomes in the following setting: Imagine we have computed $\widehat{P_n}$ by observing the responses of n over T time points and we want to use these data to predict the value of x_{nt} in period $T + 1$. If we could only make statements like: "we think that $x_{n,T+1} = i$", then our best guess would be to choose the i for which $p_n(i)$ is the highest. Then the probability of making a wrong

Figure 1: Responses to a categorical question across time: Two individual examples



guess *despite our best effort* would be $1-p_n(i)$. Clearly, under minimum mobility our guess would be failsafe correct. On the other extreme, it is easy to realize that the probability of being wrong in this setting, despite the best possible prediction, is maximized if and only if maximum mobility holds.[1]

Since we are considering mobility as instability, unpredictability, or diversity, we can order probability distributions in terms of their degree of mobility, using the concepts of the inequality literature, in particular uniform majorization (Marshall, Olkin, and Arnold, 2010) and the traditional Pigou-Dalton transfer, but applied to the domain of discrete probability distributions. We also define the following mobility orderings: the strong mobility ordering $\succ$, such that $P_X \succ P_Y$ reads "distribution X is more mobile than distribution Y". The weak mobility ordering $\succeq$, such that $P_X \succeq P_Y$ reads "distribution X is at least as mobile as distribution Y". Finally the indifference mobility ordering $\sim$ such that $P_X \sim P_Y$ reads "distributions X and Y are equally (im)mobile". Then we define a probability Pigou-Dalton transfer:

Definition 3. *Consider probabilities $p_n(i)$ and $p_n(j)$, such that $p_n(i) > p_n(j)$. A probability Pigou-Dalton transfer (PPD) is a rank-preserving transfer of probability mass δ from the*

[1]Admittedly, we could also base our prediction on different rules. For example, we could study the time trend and see if, for instance, only over the last few time points the individual has been repeating values. Then we could predict that in $T+1$ the individual will report the most recent values. But this level of analysis would require looking for structural breaks in the trend, and the like. In this first part of the paper, we are proposing a more straightforward assessment of mobility with categorical variables in which the degree of mobility is related to the proportions of time spent in each category, irrespective of the actual trajectory. By contrast, in the second part, when we introduce our concept of individual ordinal mobility, path dependence of responses will be a key feature.

greater to the lower probability, that is: $p_n(i) - \delta \geqslant p_n(j) + \delta$.

We consider that a PPD should strictly increase mobility as diversity/instability, i.e. if distribution Y is obtained from distribution X by a PPD then: $P_Y \succ P_X$. More generally, we define a uniform majorization:

Definition 4. *Distribution Y is obtained from a uniform majorization of X if $P_Y = BP_X$ where B is a bi-stochastic matrix.*

We consider that a uniform majorization should not decrease mobility as diversity, i.e. if distribution Y is obtained from distribution X by a uniform majorization then: $P_X \prec P_Y$.

Now note the following two details. First, we say that a uniform majorization should not decrease mobility instead of asserting that it increases mobility strictly. This is because the domain of bi-stochastic matrices admits matrices whose elements are only either ones or zeroes, i.e. the identity matrix and its permutations. When multiplied by any of these matrices, the probabilities of X are not changed. At best they are reshuffled. In such circumstances we do not consider mobility as diversity to have increased. Second, as is well known, any PPD can be implemented via a uniform majorization, choosing the appropriate bi-stochastic matrix.

2.3 Individual mobility indices

We define an individual mobility index: $\mathcal{M}_n : P_n \subset \mathbb{R}_+^S \to [0,1] \subset \mathbb{R}$. We consider indices which are twice continuously differentiable across the whole domain. The individual mobility index should fulfill the following desirable properties:

Axiom 1. *Maximum mobility: $\mathcal{M}_n = 1$ if and only if the individual exhibits maximum mobility.*

Axiom 2. *Minimum mobility: $\mathcal{M}_n = 0$ if and only if the individual exhibits minimum mobility.*

Axiom 3. *Sensitivity to PPD: If Y is obtained from X by PPD, then $\mathcal{M}_Y > \mathcal{M}_X$.*

Axiom 4. *Sensitivity to uniform majorization: If Y is obtained from X by uniform majorization, then $\mathcal{M}_Y \geqslant \mathcal{M}_X$.*

Considering these axioms, we identify a class of individual mobility indices fulfilling them, according to proposition 1:[2]

Proposition 1. *The twice continuously differentiable individual mobility index for multiple periods and categorical variables, $\mathcal{M}_n$, fulfills the above four axioms (1, 2, 3, and 4) if and only if it is an additively separable, symmetric, and concave function* mapping from discrete probability distributions*, and it is normalized so that $\mathcal{M}_n = 1$ only whenever $p_n(1) = p_n(2) = ... = p_n(S)$ (maximum mobility) and $\mathcal{M}_n = 0$ only whenever $\exists i | p_n(i) = 1$ (minimum mobility).*

[2]Note that fulfillment of axiom 4 implies fulfillment of axiom 3. Likewise, one can show that the two situations of maximum and minimum mobility are the only ones consistent with axiom 3.

Proof. See Appendix C. ■

Examples of $\mathcal{M}_n$ fulfilling the above axioms include:

$$\mathcal{M}_n = 1 - \frac{\frac{1}{S}\sum_{i=1}^{S}[Sp_n(i)]^{\alpha} - 1}{S^{\alpha-1} - 1} \quad \forall \alpha > 1 \tag{2}$$

$$\mathcal{M}_n = 1 - \frac{1}{2(S-1)}\sum_{i=1}^{S}\sum_{j=1}^{S}|p_n(i) - p_n(j)| \tag{3}$$

Note that the index in 3 is a Gini index mapping from the discrete probability distribution. Therefore we can similarly devise other indices with so-called rank-dependent functional forms. Meanwhile, the indices in 2 are basically members of a generalized entropy family which also map from the probability vector (instead of incomes, etc.); i.e., examples of equally available rank-independent measures. Interestingly, when $\alpha = 2$ in 2, then $\mathcal{M}_n$ is a function of the Simpson index:

$$\mathcal{M}_n = \frac{S}{S-1}\sum_{i=1}^{S} p_n(i)[1 - p_n(i)] \tag{4}$$

2.4 Quasi-orderings

Clearly, several indices fulfill the above axioms (1, 2, 3, and 4), while the orderings of individuals produced by each of them may not always coincide. This is a situation akin to that encountered in inequality measurement and other distributional fields. However, given the similarities between traditional inequality measurement and the mobility-as-diversity measurement proposed in this section, we can also identify situations in which individual mobility rankings will be robust to any choice of reasonable mobility index. This quasi-ordering applies to discrete probability distributions for variables *with the same number of categories*, and can be derived using probability Lorenz curves, $L_n(i) : [0,1]^S \rightarrow [0,1]$ which are defined the following way:

Definition 5. $L_n(i) = \sum_{j=1}^{i} r_n(j), \;\; i \in [1, S]$, *where the probabilities, r_n, are the probabilities p_n ranked from the lowest to the highest value (so that for instance, $r_n(1) = \min[p_n(1), p_n(2), ..., p_n(S)]$).*

The quasi-ordering relies on the following proposition:

Proposition 2. $\mathcal{M}_X > \mathcal{M}_Y$ *for any mobility index of the form $\mathcal{M}_n$ fulfilling the axioms 3 and 4 if and only if $L_X(i) \geqslant L_Y(i) \;\; \forall i \in [1,S]$ and $\exists j \in [1,S] | L_X(j) > L_Y(j)$.*

Proof. See Appendix C. ■

Finally, consider X and Y in proposition 2, and let X be characterized by maximum mobility, while Y exhibits strictly less than maximum mobility. Then if we plot both probability Lorenz curves, we will find that $L_X(i) \geqslant L_Y(i) \;\; \forall i \in [1,S]$ and $\exists j \in [1,S] | L_X(j) > L_Y(j)$. Therefore $\mathcal{M}_X > \mathcal{M}_Y$ for any mobility index satisfying the transfer properties and for any Y not characterized by maximum mobility (with X showing maximum mobility). Now let Y be characterized by minimum mobility, while X exhibits strictly more than minimum mobility. Then, again, if we plot both probability Lorenz curves, we will find that $L_X(i) \geqslant L_Y(i) \quad \forall i \in [1,S]$ and $\exists j \in [1,S] | L_X(j) > L_Y(j)$, which leads to $\mathcal{M}_X > \mathcal{M}_Y$ for any mobility index satisfying the transfer properties and for any X not characterized by minimum mobility (with Y showing minimum mobility). Hence, mobility indices satisfying the transfer properties always yield extreme values corresponding to these two benchmark situations and only to them.

This detail is important, among other things, in order to rule out the suitability of other indices of ordinal variation for our purpose of measuring multiple-period mobility with the meaning of instability/unpredictability. For instance, take the Index of Ordinal Variation (IOV) by Berry and Mielke (1992). The IOV's minimum value coincides with a situation of no ordinal inequality, which in our context occurs only when there is minimum mobility. However the IOV's maximum value is attributed to a situation where half the population is in the bottom category and the other half is in the top category, where the categories correspond to an ordinal variable. By contrast, such a situation would not coincide with our notion of maximum mobility because one could induce further mobility by performing PPD transfers between the extreme categories and the intermediate (initially empty) categories. In other words, the IOV does not fulfill our transfer axioms; therefore it is not useful as a measure of mobility in terms of multiple-period instability. Other indices from the growing literature on ordinal inequality (see Abul Naga and Yalcin, 2010, Apouey and Silber, 2013, Lv, Wang, and Xu, 2015, for some recent examples) face the same problem. Unfortunately, as shown below, these indices are not useful for the measurement of multi-period mobility as average traveled distance either.

2.5 Normalization issues when the time period is small relative to the number of categories

The above indices work well in theory. However in practice we need to compute the probabilities in 1 from the data. The situations of minimum mobility are easy to spot empirically as they only require one probability to be equal to one. By contrast, when the time period is short, the appearance of the probability distribution under a situation of maximum mobility depends on the relationship between S and T. Then, for any chosen individual mobility index, the maximum value varies accordingly. Therefore when the time period is short, mobility indices should be adjusted according to the following formula:

$$\mathcal{A}_n = \frac{\mathcal{M}_n - \min \mathcal{M}_n}{\max \mathcal{M}_n - \min \mathcal{M}_n} \tag{5}$$

When the time period is relatively short, there are three cases of maximum mobility:

Case 1. *If* $T \leqslant S$*:* $p_n(i) = \frac{1}{T} \; \forall i \in [1,T]$ *and* $p_n(i) = 0 \; \forall i \in [T+1,S]$.

Case 2. *If* $T > S$ *and* $T = \lambda S$ *where* $\lambda \in \mathbb{N}_{++}$*:* $p_n(i) = \frac{1}{S} \; \forall i \in [1,S]$.

Case 3. *If* $T > S$ *and* $T = \lambda S + R$ *where* $\lambda, R \in \mathbb{N}_{+}$ *and* $R < S$*:* $p_n(i) = \frac{\lambda+1}{T} \; \forall i \in [1,R]$ *and* $p_n(i) = \frac{\lambda}{T} \; \forall i \in [R+1,S]$.

Consider, for example, the adjustment to the individual mobility index based on the Simpson index, as in 4:

$$\text{Case 1: } \mathcal{A}_n = \frac{T}{T-1} \sum_{i=1}^{S} p_n(i)[1 - p_n(i)] \tag{6}$$

$$\text{Case 2: } \mathcal{A}_n = \frac{S}{S-1} \sum_{i=1}^{S} p_n(i)[1 - p_n(i)] \tag{7}$$

$$\text{Case 3: } \mathcal{A}_n = \frac{T^2 \sum_{i=1}^{S} p_n(i)[1 - p_n(i)]}{R[T - 2\lambda - 1] + S\lambda[T - \lambda]} \tag{8}$$

Now consider the adjustment to the individual mobility index based on the Gini index, as in 3:

$$\text{Case 1: } \mathcal{A}_n = \frac{S-1}{T-1} - \frac{1}{2(T-1)} \sum_{i=1}^{S} \sum_{j=1}^{S} |p_n(i) - p_n(j)| \tag{9}$$

$$\text{Case 2: } \mathcal{A}_n = 1 - \frac{1}{2(S-1)} \sum_{i=1}^{S} \sum_{j=1}^{S} |p_n(i) - p_n(j)| \tag{10}$$

$$\text{Case 3: } \mathcal{A}_n = \frac{T(S-1) - \frac{T}{2} \sum_{i=1}^{S} \sum_{j=1}^{S} |p_n(i) - p_n(j)|}{T(S-1) - R(S-R)} \tag{11}$$

3 Multiple-period mobility as average distance traveled across categories

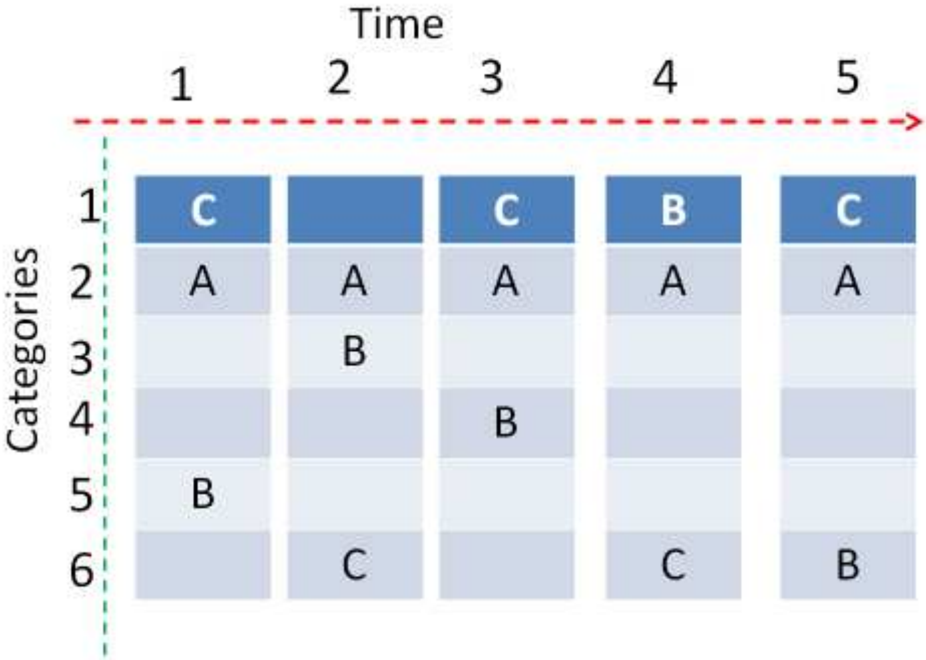
3.1 Preliminaries and notation

For the assessment of mobility as average traveled distance we, firstly, define the normalized "distance" transited by individual n between periods t and $t-1$ by: $d_{nt} = \frac{|x_{nt} - x_{n,t-1}|}{S-1}$. The statistic d_{nt} measures the number of categories "jumped" between the two periods as a proportion of the maximum transit possible, i.e. moving from one extreme category to the other one ($S-1$). We also define the vector of all distances traveled across transitions: $D_n := (d_{n2}, d_{n3}, ..., d_{nT})$.

3.2 A concept of individual mobility specifically for ordinal variables

With an ordinal variable for each individual over several time points we can ask again: how stable is the individual's experience according to the variable? Does the individual always exhibit the same value? However, unlike the case of unordered categories, the unpredictability situation does not seem to be the only appropriate or useful benchmark of maximum mobility, because now the order of the categories provides us with a limited notion of distance. Hence we could posit, for example, that someone who is constantly moving between bottom and top categories exhibits more mobility than someone who visits any category randomly (yielding a uniform distribution), since the latter person will be, *on average*, "travelling" a shorter distance.

Figure 2: Responses to an ordinal question across time: Three individual examples



For example, figure 2 shows the response patterns of individuals "A", "B", and "C" to a life satisfaction question with six categories as possible answers, over five time points. Individual "A" always answers category "2", whereas individual "B" answers different categories in every year, and individual "C" answers either the lowest or the highest category roughly evenly. While the pattern of "A" is stable, and "B" exhibits a highly unstable pattern which is difficult to predict, "C" shows a pattern whose predictability lies somewhere between "A" and "B", but manifests a higher average movement across categories between any given pair of consecutive years. As before, it is interesting to learn the extent to which these different patterns can be explained by similar fluctuations (or lack thereof) in the events of individuals' lives; including whether the extreme cases reflect appraisals of life experiences or bad respondent behaviour.

3.3 The problem with using ordinal inequality measurement to operationalise the notion of individual multiple-period mobility with ordinal variables

With the above in mind, we reinstate definition 1 as our benchmark for minimum mobility as movement across ordered categories (i.e. an individual exhibits minimum mobility as movement by staying in the same position throughout).

Just as we borrowed from the traditional inequality literature in order to measure individual mobility as diversity of responses, we are tempted to rely on the more recent literature on ordinal-variable inequality in order to measure individual multiple-period mobility with ordinal variables. Some recent examples of indices and related quasi-orderings can be found, inter alia, in Reardon (2009), Abul Naga and Yalcin (2010), Apouey and Silber (2013), Lv et al. (2015). Unfortunately, as we show in this subsection, this measurement framework is not satisfactory for the purpose.

In the ordinal inequality framework minimum inequality occurs when everybody reports the same category. Hence, translated to a mobility setting, this benchmark would correspond to our definition 1. On the other extreme, the benchmark of maximum ordinal inequality occurs when half of the population is in the lowest category while the other half reports the highest category. If we translate this benchmark into a notion of maximum mobility we obtain:

Definition 6. *Maximum ordinal mobility: An individual exhibits maximum ordinal mobility if* $p_n(1) = p_n(S) = 0.5$.

According to definition 6, maximum ordinal mobility occurs when the individual probability distribution is bimodal with support only on the two extreme ordered categories. As figure 3 shows, this same bimodality can be obtained in different ways. For example, individual "A" is always moving between the two extremes from year to year. Thus he spends about 50% of the time in each extreme category. Individual "B" spends the same proportion of time in each of the same two categories. However, individual "B" spends most of the time *not changing categories at all*. Only once, between periods 2 and 3, does he switch from one extreme to the other. In this context, any mobility index adapted from the ordinal-inequality measurement framework would fail to rank "A" and "B" differently. Yet intuitively we would be inclined to state that "A" exhibits higher mobility than "B". For one thing, "A" travels a longer normalized distance between categories on an average transition between two periods. In fact, $d_{At} = 1,\ t = 2,3,4,5$, whereas $d_{Bt} = 0,\ t = 2,4,5 \wedge d_{B3} = 1$. Consequently the recent ordinal-inequality measurement framework is not suitable for measuring mobility with ordinal variables over several periods.

3.4 A proposal for ordinal variables based on transited distances

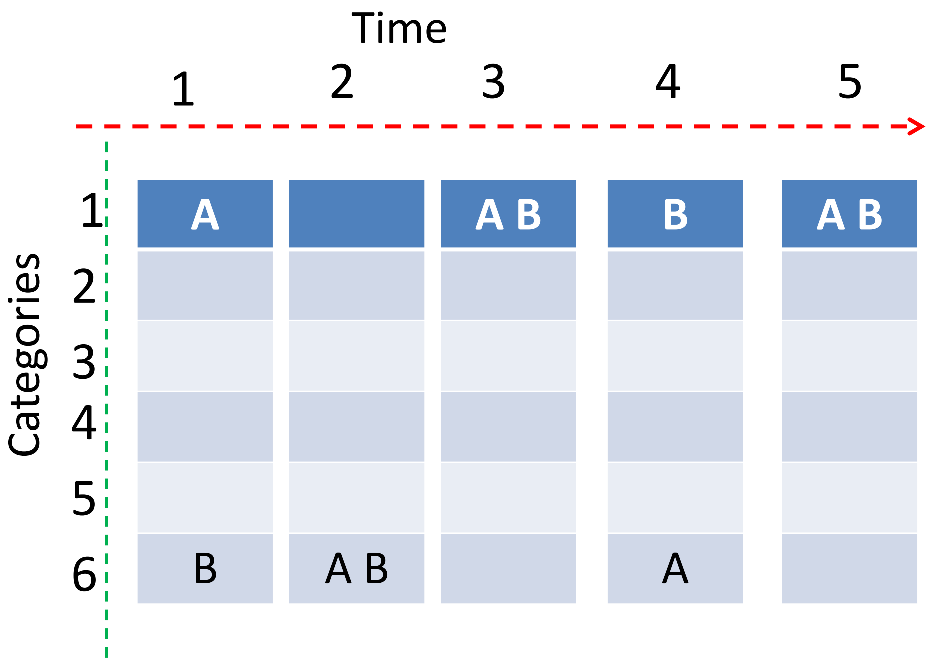
Bartholomew (1982) proposed several mobility indices for transition matrices. Let a typical conditional probability from a transition matrix be defined by: $p(i|j) \equiv \Pr[x_t = i|x_{t-1} = j]$. Then one of the indices proposed by Bartholomew (1982) was:

$$B = \sum_{i=1}^{S} p^*(i) \sum_{j=1}^{S} p(i|j)|i-j|, \tag{12}$$

where the probabilities, $p^*(i)$, correspond to the ergodic distribution. Shorrocks (1978b) showed that replacing each $p^*(i)$ by weights independent from the transition matrix (e.g. the probability distribution in period $t-1$ or plainly $\frac{1}{S}$) renders B in fulfillment of several desirable properties consistent with the notion of mobility as movement (Van de Gaer et al., 2001). Intuitively, B measures a social average of the distances traveled between $t-1$ and t, i.e. $|i-j|$.

Following a similar reasoning, we note that in a multiple-period setting, each individual n makes $T-1$ trips in T time points. Between t and $t-1$ individual n travels, in

Figure 3: Extreme responses to an ordinal question across time: Two individual examples



fact, a normalized distance of d_{nt}. So we could also compute mobility indices that capture some form of average distance traveled. In order to work with the statistics d_{nt} we need to acknowledge that we are only quantifying the numbers of categories traveled away from some initial category. Implicitly we assume that any distance between two adjacent categories in any given transition period is neither longer nor shorter than any other distance between any other two adjacent categories in any other transition period.

With these assumptions in place, we define the following benchmark of minimum and maximum multiple-period mobility for ordinal variables:

Definition 7. *Minimum ordinal mobility: An individual exhibits minimum ordinal mobility if and only if* $d_{nt} = 0 \ \forall t = 2, ..., T$.

Note that definition 7 is identical to our previous definition of minimum mobility for the concept of mobility as unpredictability.

Definition 8. *Maximum ordinal mobility: An individual exhibits maximum ordinal mobility if and only* $d_{nt} = 1 \ \forall t = 2, ..., T$.

Note now that definition 8 is fundamentally different from definition 6. When individual n is in the situation described by 8 he will also feature $p_n(1) = p_n(S) = 0.5$, i.e. the situation of 6. *However the reverse is not true.*

Before we define an index for the multiple-period, ordinal individual mobility, it is worth introducing a mobility-inducing transformation in this context. Note that every x_{nt}, except those at the period extremes (i.e. x_{n1} and x_{nT}) is involved in two distances, namely d_{nt} and $d_{n,t+1}$. In this context we define a distance-increasing-jump (DIJ):

Definition 9. *Y is obtained from X through a distance-increasing-jump (DIJ) if* $y_{nt} = x_{nt}$ $\forall t \neq \tau$ *and: (1)* $y_{n,\tau} < x_{n\tau} \leqslant \min(x_{n,\tau-1}, x_{n,\tau+1})$*; or (2)* $y_{n,\tau} > x_{n\tau} \geqslant \max(x_{n,\tau-1}, x_{n,\tau+1})$*; or (3)* $\min(x_{n,\tau-1}, x_{n,\tau+1}) \leqslant x_{n\tau} \leqslant \max(x_{n,\tau-1}, x_{n,\tau+1})$ *but either* $y_{n,\tau} < \min(x_{n,\tau-1}, x_{n,\tau+1})$ *or* $y_{n,\tau} > \max(x_{n,\tau-1}, x_{n,\tau+1})$.

In short, definition 9 describes the only three possible ways in which, through one single DIJ involving two distances, the average distance traveled in Y will be rendered unequivocally greater than in X. These three ways correspond to the only three possible scenarios in which the distances $d_{n\tau}$ and $d_{n,\tau+1}$ would both increase simultaneously thereby increasing any measure of average distance traveled unambiguously. In the case of the period extremes we complement definition 9 with the following definition:

Definition 10. *Y is obtained from X through a distance-increasing-jump on the extremes (DIJE) if* $y_{nt} = x_{nt}$ $\forall t \neq (1, T)$ *and: (1)* $|y_{n1} - x_{n2}| > |x_{n1} - x_{n2}|$*; or (2)* $|y_{nT} - x_{n,T-1}| > |x_{nT} - x_{n,T-1}|$.

We now define an individual mobility index: $\mathcal{B}_n : D_n \subset \mathbb{R}_+^{S-1} \to [0,1] \subset \mathbb{R}$. The individual mobility index should fulfill the following desirable properties:

Axiom 5. *Maximum mobility:* $\mathcal{B}_n = 1$ *if and only if the individual exhibits maximum mobility, i.e.* $d_{nt} = 1$ $\forall t = 2, ..., T$.

Axiom 6. *Minimum mobility:* $\mathcal{B}_n = 0$ *if and only if the individual exhibits minimum mobility, i.e.* $d_{nt} = 0$ $\forall t = 2, ..., T$.

Axiom 7. *Sensitivity to DIJ and DIJE: If Y is obtained from X by DIJ and/or DIJE, then* $\mathcal{B}_Y > \mathcal{B}_X$.

Finally, we do not want mobility measurement to be affected by permutations of the elements of the distance vector D_n, which otherwise render all distances the same. If we allowed these permutations to affect the value of the mobility index, then for instance, we would change our mobility judgment if we reversed the time sequence (e.g. with the first time point becoming the last one, the second-to last becoming the second point, and so forth). We state this desired insensitivity to permutations of distances with the following axiom:

Axiom 8. *Symmetry: If* $D_Y = D_X Q$ *where Q is a $T-1$-dimensional, square permutation matrix, then* $\mathcal{B}_Y = \mathcal{B}_X$.

Considering these axioms, we identify a class of individual mobility indices fulfilling them, according to proposition 2:

Proposition 3. *The individual mobility index for multiple periods and ordinal variables, $\mathcal{B}_n$ fulfills the above four axioms (5, 6, 7, 8) if and only if it is a strictly increasing function of an additively decomposable, symmetric function* mapping from distance space*, strictly increasing in every element of D_n, and it is normalized so that* $\mathcal{B}_n = 1$ *only whenever* $d_{nt} = 1$ $\forall t = 2, ..., T$ *and* $\mathcal{B}_n = 0$ *only whenever* $d_{nt} = 0$ $\forall t = 2, ..., T$.

Proof. See Appendix C. ■

Essentially, proposition 3 describes the following class:

$$\mathcal{B}_n = h[\sum_{t=2}^{T} g(d_{nt})], \text{ with } h' > 0,\ g' > 0,\ h[\sum_{t=2}^{T} g(0)] = 0,\ h[\sum_{t=2}^{T} g(1)] = 1 \tag{13}$$

For instance, members of 13 would include some generalized means (e.g. the arithmetic mean, the Euclidean mean, but not the geometric mean). In the case of the arithmetic mean we could have:

$$\mathcal{B}_n = \frac{1}{T-1}\sum_{t=2}^{T} d_{nt} \tag{14}$$

3.5 Quasi-orderings

As observed with the case of mobility as unpredictability, several indices fulfill the axioms for mobility as distance traveled (5, 6, 7), while the orderings of individuals produced by each of them may not always coincide. However, we can also identify situations in which mobility-as-distance rankings will be robust to any choice of a reasonable mobility index. Similar to the previous quasi-ordering, this one also applies to discrete probability distributions for variables *with the same number of categories*. In order to derive the pre-ordering condition we rely upon a well-established result of first-order stochastic dominance for discrete variables. Let $\phi_n(i) \equiv \Pr[d_{nt} = i]$ and $\Phi_n(i) \equiv \Pr[d_{nt} \leqslant i] = \sum_{j=0}^{i} \phi_n(j)$. Then the quasi-ordering relies on the following proposition:

Proposition 4. *$\mathcal{B}_n(X) > \mathcal{B}_n(Y)$ for any $\mathcal{B}_n$ of the form 13 if and only if $\Phi_X(i) \leqslant \Phi_Y(i)\quad \forall i \in [0, \frac{1}{S-1}, \frac{2}{S-1}, ..., 1]$ and $\exists j \in [0, \frac{1}{S-1}, \frac{2}{S-1}, ..., 1] | \Phi_X(j) < \Phi_Y(j)$.*

Proof. See Appendix C. ∎

4 Social mobility indices

We can construct social mobility indices by aggregating individual mobility indices, similar to the poverty and wellbeing measurement literature. The most natural proposal is the arithmetic average:

$$\mathcal{M} = \frac{1}{N}\sum_{n=1}^{N} \mathcal{M}_n \tag{15}$$

The index in 15 fulfills the well-known properties of symmetry, population replication invariance, additive decomposability and subgroup consistency. It is also the case that: $\mathcal{M} = 0 \leftrightarrow \mathcal{M}_n = 0\ \forall n$ and $\mathcal{M} = 1 \leftrightarrow \mathcal{M}_n = 1\ \forall n$.

5 Empirical illustration: Individual mobility of subjective wellbeing responses in the United Kingdom

In this section we present estimates of the mobility indices proposed in the previous sections, and illustrate their usefulness with an application to life satisfaction measures from the British Household Panel Survey (BHPS). Several international datasets provide longitudinal data on household characteristics and qualitative data. Among these, the BHPS features high-quality ordinal data on self-reported levels of life satisfaction and happiness for all years covered.

The BHPS follows the same representative sample of individuals over a period of 18 years from 1991 to 2009. Each annual interview round is called a wave. In our study we use 18 waves of data, where each wave is principally household-based, interviewing every adult member of sampled households. Each wave consists of over 5,500 households and over 10,000 individuals drawn from 250 areas of Great Britain. Samples of 1,500 households from Scotland and Wales (3,000 in total) were added to the main sample in 1999; later in 2001 a sample of 2,000 Northern Ireland households was also added.

Our main variables of interest are four separate subjective wellbeing indicators. These are: general happiness, life satisfaction, and two subjective well-being variables based on the Likert and Caseness algorithms, respectively. The General Happiness variable (*ghq*) features responses graded on a decreasing level of happiness, 1 to 4. Life satisfaction is graded on a scale of 1 to 7, with increasing levels of life satisfaction for increasing values of the variable. The Likert-based variable has 36 categories, while the Caseness-based variable has 12 categories. Both represent decreasing scales of happiness for increasing values of the variable.

We have at least 3200 observations for each model estimated in Tables 1 to 4. The dataset used for each regression is balanced, i.e., there are no missing observations for each of the datasets.[3] The number of observations for each estimated model varies, as each regression sample is chosen such that it has full yearly availability of the socio-economic characteristics.

We estimate the mobility indices based on the Gini and Simpson indices as described in equations 3 and 4, together with two members from the class of mean distance functional forms in equation 14: the arithmetic mean and the Euclidean mean. Depending upon the number of categories of the life satisfaction indicators, we apply the normalisation procedure as discussed in Section 2.5 for the corresponding indices. For ease of discussion, we name the indices 3 and 4 the Gini mobility index and the Simpson mobility index respectively, while the two indices based on 14, are called the Arithmetic mean distance measure and the Euclidean mean distance measure.[4]

We now illustrate how one may use the estimated measures of mobility with an empirical exercise popular in the subjective wellbeing literature. We examine the socio-economic determinants of mobility using ordinal subjective wellbeing measures as a proxy of one's wellbeing status. For that, we estimate the following relationship with ordinary least squares:

[3]Mobility indices were also estimated over different wave periods: $3-17, 1-12, 9-18$, in order to observe different mobility patterns. For the shorter wave periods, the number of observations for each wave was at least 4500, and for the shortest time period 9-18, we have over 6000 observations.

[4]For each of the life satisfaction variables, the normalisation procedure from Section 2.5 has been applied. For the variables general happiness, life satisfaction, and subjective well-being (Caseness), Case 3 applies, and therefore indices 8 and 11 were estimated. For the variable subjective wellbeing (Likert), Case 1 applies, and therefore indices 6 and 9 were estimated for the Simpson and Gini indices.

$$MobilityIndex_i = \alpha + \mathbf{X}_i\beta + \epsilon_i \quad (16)$$

where, $MobilityIndex_i$ is the mobility index of subjective wellbeing measured for individual i, $\mathbf{X}_i$ is a vector of socio-economic variables described below, and ϵ_i is assumed to be normally distributed, $N(0, \sigma_\epsilon^2)$. Tables 1 to 4 present the results of the estimates of the model in 16 using the the Gini mobility index, the Simpson mobility index and the Arithmetic and Euclidean mean distance measures as mobility measures, estimated with the four subjective wellbeing variables available in the BHPS: general happiness, life satisfaction, and two subjective wellbeing indices: Likert and Caseness (details of each variable is in the Appendix). We use "shock" variables, rather than just levels, for events that are likely to induce changes in happiness and other subjective wellbeing indicators. We also employ changes in status for variables such as unemployment, illness and divorces. For unemployment, we focus on three types of changes in one's status of being unemployed - first, whether unemployed or not, in the wave period under study, second, number of changes in unemployment status within the past 26 weeks, and finally, number of changes in unemployment status in the last 52 weeks. Illness is measured by the number of visits made to the GP in the given year. Divorce is measured by whether the respondent had been divorced in the given time period. We also estimated the number of divorces over the entire time period, and the results obtained in the tables below using this latter variable are identical to the former's, hence we dropped it from the analysis.

In addition to the above "shock" variables, we have also standard socio-economic variables, such as gender, age, education, income, number of children in household and marital status. The education variable has been rescaled to reflect increasing number of years in education.

Each column in the tables presents results of an OLS regression where the individual mobility index was regressed upon the socio-economic variables in any one particular wave. Remarkable stability is observed in the relationships estimated for each of the the waves' estimates. The results presented in the table are representative for wave 10. The choice of the waves is ad-hoc, simply meant to represent an "average year" in the time period of 1991 to 2009 equally. Estimates using all other waves are available from the authors on request.

Other strategies were also adopted, for example, using the averages of the variables over the entire time period. For this, several difficulties were encountered. First, the interpretation of several of the variables (such as education or marital status) were lost. Second, all waves do not have the same number of respondents on these questions and finally, the questionnaire changed and therefore the categorisation and coding of some of the variables changed as well (for example, marital status). Nevertheless, we estimated a model with averages of the socio-economic variables after having taken all of the above considerations into account, and the results are identical to those that we observe with the wave-specific setting. Estimates are available from the authors on request.

Finally, in order to increase the number of observations per model, we also chose subsets of years, such as 3-12, 9-18, 1-12. The estimates obtained are very similar to those we present below and are again available from the authors on request. Each of the four tables

below present the regression results for the four individual subjective-wellbeing variables separately: general happiness, life satisfaction, subjective wellbeing Likert, and subjective wellbeing Caseness.

Table 1: General Happiness

	Gini	Simpson	Arithmetic	Euclidean
Female	0.052***	0.072***	0.077***	0.119***
	(0.007)	(0.010)	(0.010)	(0.016)
Illness	0.024***	0.033***	0.039***	0.055***
	(0.003)	(0.004)	(0.004)	(0.007)
Married	-0.017**	-0.003	-0.042***	-0.071***
	(0.008)	(0.011)	(0.010)	(0.017)
Was unemployed	0.017	0.042**	0.043	0.059
	(0.018)	(0.023)	(0.031)	(0.053)
Unemployed for 26 weeks	0.021	0.029	-0.019**	0.041**
	(0.024)	(0.023)	(0.009)	(0.016)
Unemployed for 52 weeks	0.011	-0.036	-0.022*	-0.047**
	(0.012)	(0.049)	(0.012)	(0.020)
Age	-0.003***	-0.006***	0.005***	0.007***
	(0.000)	(0.000)	(0.000)	(0.001)
Education level	0.005***	0.008***	-0.007***	0.010***
	(0.001)	(0.001)	(0.001)	(0.002)
Log annual income	0.006**	0.004**	0.012***	-0.017**
	(0.003)	(0.005)	(0.005)	(0.008)
Number of children	0.003	0.000	0.004***	0.010***
	(0.004)	(0.000)	(0.005)	(0.002)
Divorced	0.000	0.081***	0.030*	0.001*
	(0.016)	(0.022)	(0.015)	(0.000)
Constant	0.266***	0.525***	0.388***	0.469***
	(0.030)	(0.051)	(0.055)	(0.088)
Observations	3237	3237	3252	3252
Adjusted R-squared	0.142	0.145	0.152	0.126
Notes	Robust standard errors in parentheses			
	*** p<0.01, ** p<0.05, * p<0.1			

For all four subjective wellbeing indicators, being female is positively and significantly associated with all four estimated mobility indices. Similarly, being ill (measured by the number of visits to a GP) is also positively and significantly associated with increasing mobility. Being unemployed also is significantly associated with mobility in happiness levels with some variations by index and by the measure of unemployment used. Of the three measures of unemployment used, being unemployed for 26 weeks is more frequently significantly associated with mobility compared with being unemployed for 52 weeks, or just being unemployed. Age is consistently observed to be negatively and (mostly) significantly associated with mobility. In other words, the young have significant changes in their levels of happiness compared with the old. We also find that barring the case of the life satisfaction measure of happiness in Table 2 education (measured in number of years) is associated with higher mobility in happiness levels. Number of children is not found to be stable in its association with mobility in these regressions, but this result is to change when we focus on high and low mobility subsamples in the following section.

It is worth noting that the relationships between each socio-economic variable and the

Table 2: Life Satisfaction

	Gini	Simpson	Arithmetic	Euclidean
Female	0.022***	0.055***	0.155***	0.061***
	(0.005)	(0.010)	(0.035)	(0.012)
Illness	0.016***	0.025***	0.129***	0.048***
	(0.002)	(0.004)	(0.017)	(0.006)
Married	-0.022***	-0.034***	-0.248***	-0.086**
	(0.006)	(0.010)	(0.042)	(0.014)
Was unemployed	0.009	-0.033***	0.072	0.047
	(0.0135)	(0.012)	(0.128)	(0.038)
Unemployed for 26 weeks	0.049**	0.042**	-0.187***	0.062**
	(0.024)	(0.021)	(0.047)	(0.013)
Unemployed for 52 weeks	-0.008	0.017	-0.128*	-0.049**
	(0.027)	(0.011)	(0.067)	(0.020)
Age	-0.000	-0.000	0.002*	0.000
	(0.000)	(0.000)	(0.001)	(0.000)
Education	-0.004***	-0.007***	-0.029***	-0.011
	(0.000)	(0.001)	(0.006)	(0.002)
Log annual income	-0.002	-0.002	0.054***	-0.014**
	(0.002)	(0.004)	(0.018)	(0.006)
Number of children	0.013***	0.022***	0.090***	0.035***
	(0.003)	(0.005)	(0.020)	(0.007)
Divorced	0.023**	0.025**	0.013**	0.021*
	(0.12)	(0.013)	(0.005)	(0.010)
Constant	0.245***	0.773***	1.389***	0.737***
	(0.021)	(0.042)	(0.192)	(0.065)
Observations	3560	3560	3237	3237
Adjusted R-squared	0.065	0.067	0.090	0.081
Notes	Robust standard errors in parentheses			
	*** p<0.01, ** p<0.05, * p<0.1			

Table 3: Subjective Wellbeing, Likert

	Gini	Simpson	Arithmetic	Euclidean
Female	0.020***	-0.008***	0.620***	8.752***
	(0.003)	(0.001)	(0.072)	(1.117)
Illness	0.017***	0.007**	0.338***	4.144***
	(0.001)	(0.000)	(0.030)	(0.491)
Married	-0.009**	0.001***	-0.242***	-3.526***
	(0.004)	(0.000)	(0.077)	(1.207)
Was unemployed	0.011**	0.004	0.320	5.647
	(0.005)	(0.006)	(0.266)	(4.771)
Unemployed for 26 weeks	-0.009	-0.001	0.279***	4.509***
	(0.026)	(0.013)	(0.069)	(1.167)
Unemployed for 52 weeks	0.014	-0.000	-0.294***	-4.650***
	(0.029)	(0.015)	(0.094)	(1.520)
Age	-0.002***	-0.001***	-0.030***	-0.359***
	(0.000)	(0.000)	(0.003)	(0.042)
Education	0.002***	0.001**	0.056***	0.777***
	(0.000)	(0.000)	(0.010)	(0.156)
Log annual income	0.002	0.002**	0.032	0.193
	(0.001)	(0.001)	(0.035)	(0.552)
Number of children	0.000	0.000	0.060	0.829
	(0.000)	(0.000)	(0.040)	(0.675)
Divorced	0.017**	0.006**	0.003	0.013
	(0.007)	(0.003)	(0.030)	(0.014)
Constant	0.213***	0.376***	3.582***	28.675***
	(0.018)	(0.009)	(0.398)	(6.261)
Observations	3003	3003	3003	3003
Adjusted R-squared	0.137	0.089	0.153	0.113
Notes	Robust standard errors in parentheses			
	*** p<0.01, ** p<0.05, * p<0.1			

Table 4: Subjective Wellbeing, Caseness

	Gini	Simpson	Arithmetic	Euclidean
Female	0.027***	0.075***	0.481***	3.556***
	(0.004)	(0.011)	(0.049)	(0.370)
Illness	0.024***	0.058***	0.248***	1.560***
	(0.002)	(0.004)	(0.020)	(0.162)
Married	-0.015***	-0.032***	-0.144***	-0.975**
	(0.005)	(0.011)	(0.051)	(0.401)
Was unemployed	0.020	0.059*	0.254	2.043
	(0.014)	(0.034)	(0.161)	(1.392)
Unemployed for 26 weeks	-0.010	0.029	0.157***	1.220***
	(0.027)	(0.069)	(0.041)	(0.345)
Unemployed for 52 weeks	-0.007	-0.058	-0.182***	-1.319***
	(0.033)	(0.077)	(0.058)	(0.468)
Age	-0.001***	-0.002***	-0.013***	-0.102***
	(0.000)	(0.000)	(0.002)	(0.014)
Education	0.002***	0.005***	0.032***	0.236***
	(0.000)	(0.001)	(0.007)	(0.054)
Log annual income	0.000	0.004	0.035	0.234
	(0.002)	(0.005)	(0.025)	(0.182)
Number of children	0.005**	0.010**	0.066**	0.449**
	(0.002)	(0.005)	(0.026)	(0.208)
Divorced	0.017**	0.038**	0.034**	0.045**
	(0.009)	(0.019)	(0.016)	(0.022)
Constant	0.112***	0.0514***	1.326***	6.823***
	(0.026)	(0.060)	(0.273)	(2.018)
Observations	3003	3003	3003	3003)
Adjusted R-squared	0.105	0.111	0.137	0.115
Notes	Robust standard errors in parentheses			
	*** p<0.01, ** p<0.05, * p<0.1			

mobility indices are remarkably stable across all tables' results. In other words, the respective relationships between the mobility index and its covariates are robust and stable for all models estimated, for all life satisfaction measures, and for all mobility indices: the Gini, Simpson and the Arithmetic and Euclidean mean distance indices.

In the following section we focus on those who are highly mobile or have very low mobility and estimate a separate set of models to identify the socio-economic characteristics of these groups.

5.1 Determinants of high mobility and low mobility

We now isolate individuals for whom the estimated mobility indices take very low and very high values. We have found that for cut-off values of the mobility index taking values less than 0.2 and for values above 0.8, the sample size becomes extremely small. For example, there are almost no observations for the mobility indices taking values 0 or 1, or even for values such as 0.05 and 0.95. We therefore selected cut-off values of 0.35 and less for low mobility and 0.65 and above for high mobility. For robustness we have also chosen values in the vicinity of the above, and the main results of the regression analysis remain unchanged.

On the basis of our definition of high mobility (if the mobility index is greater than 0.65), and low mobility (if the mobility index is less than 0.35), we define two binary variables:

(1) $low = 1$ if the individual has low mobility, and equals 0 otherwise.

(2) $high = 1$ if the individual has high mobility, and equals 0 otherwise.

We estimate the following logit model for the determinants of low mobility given below:

$$low_i = \gamma + \mathbf{Z}_i \lambda \tag{17}$$

where $low_i = 1$ if the mobility index for individual i is lower than or equal to 0.35, otherwise $low_i = 0$; $\mathbf{Z}_i$ is a vector of socio-economic characteristics (consisting of the same variables used in the previous regression models). The average marginal effects of the above estimated model are presented in Tables 5 and 6.

Table 5: Socio-economic determinants of low mobility, selected indices

	General Happiness				Life Satisfaction			
	Gini	Simpson	Arithmetic	Euclidean	Gini	Simpson	Arithmetic	Euclidean
Female	0.040***	0.015**	0.021***	-0.017***	0.039***	0.024*	0.015**	0.016**
	(0.015)	(0.007)	(0.006)	(0.006)	(0.013)	(0.014)	(0.008)	(0.008)
Illness	-0.011**	0.010***	0.009***	-0.009***	0.003	0.013**	-0.006*	0.006*
	(0.006)	(0.003)	(0.002)	(0.003)	(0.005)	(0.005)	(0.003)	(0.003)
Married	0.065***	0.002	0.010***	0.010	0.039***	0.014	0.002	0.000
	(0.017)	(0.008)	(0.006)	(0.006)	(0.014)	(0.016)	(0.008)	(0.009)
Was unemployed	0.003	0.014	-0.012	-0.008	0.029	-0.000	0.011	0.011
	(0.037)	(0.010)	(0.020)	(0.020)	(0.034)	(0.000)	(0.026)	(0.026)
Unemployed for 26 weeks	-0.005	-0.003	0.001	0.001	-0.060	0.012	-0.019**	-0.018*
	(0.050)	(0.013)	(0.006)	(0.006)	(0.043)	(0.008)	(0.009)	(0.010)
Education	0.008***	0.001	-0.001***	-0.001***	0.009***	0.002	0.001***	0.001***
	(0.002)	(0.001)	(0.000)	(0.000)	(0.002)	(0.002)	(0.000)	(0.000)
Log Income	0.025***	-0.004	0.001	0.001	0.005	0.010	0.001	0.001
	(0.006)	(0.003)	(0.001)	(0.001)	(0.006)	(0.008)	(0.001)	(0.001)
Number of children	-0.007	0.002	-0.001	-0.001	-0.016	0.000	-0.000	-0.000
	(0.008)	(0.004)	(0.003)	(0.003)	(0.007)	(0.009)	(0.004)	(0.004)
Divorced	-0.022	-0.021	-0.036	-0.071**	0.048	0.013	-0.058	-0.026
	(0.033)	(0.015)	(0.036)	(0.037)	(0.029)	(0.010)	(0.037)	(0.037)
Age	-0.001***	-0.001***	-0.001**	-0.001***	-0.001***	-0.001***	-0.001***	-0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	4397	1206	1548	1480	4397	293	745	725
Adjusted R^2	0.022	0.043	0.043	0.065	0.015	0.046	0.015	0.019
Notes:	Robust standard errors in parentheses							
	*** p<0.01, ** p<0.05, * p<0.1							

Table 6: Socio-economic determinants of low mobility, selected indices

	Subjective Wellbeing, Likert				Subjective Wellbeing, Caseness			
	Gini	Simpson	Arithmetic	Euclidean	Gini	Simpson	Arithmetic	Euclidean
Female	0.070***	-0.002	-0.084	-0.072***	0.070***	0.037***	0.027**	0.026**
	(0.015)	(0.006)	(0.166)	(0.201)	(0.015)	(0.009)	(0.011)	(0.011)
Illness	-0.002	0.009***	-0.142	0.151	-0.002	0.021***	-0.019***	0.015**
	(0.006)	(0.003)	(0.070)	(0.093)	(0.006)	(0.005)	(0.006)	(0.006)
Married	0.092***	0.004	-0.027	-0.056	0.092***	0.002	0.007	0.000
	(0.017)	(0.007)	(0.113)	(0.113)	(0.017)	(0.010)	(0.011)	(0.012)
Was unemployed	0.025	-0.025	0.000	0.000	0.025	0.000**	0.016	0.034
	(0.037)	(0.017)	(0.000)	(0.000)	(0.037)	(0.000)	(0.043)	(0.042)
Unemployed for 26 weeks	0.002	-0.011	-0.043	0.087	0.002	0.001	0.030**	0.026*
	(0.051)	(0.017)	(0.193)	(0.189)	(0.051)	(0.001)	(0.014)	(0.015)
Education	0.012***	-0.001	-0.003	-0.004	0.012***	0.001	-0.001	0.000
	(0.002)	(0.001)	(0.004)	(0.004)	(0.002)	(0.001)	(0.000)	(0.000)
Log Income	0.010	0.003	-0.004	0.010	0.032***	0.005	0.003*	0.003
	(0.016)	(0.003)	(0.015)	(0.016)	(0.006)	(0.004)	(0.002)	(0.002)
Number of children	0.013	-0.001	-0.013**	0.013	-0.004	-0.003	-0.004	0.001
	(0.056)	(0.003)	(0.043)	(0.056)	(0.008)	(0.005)	(0.006)	(0.006)
Divorced	0.073**	0.023**	-0.024	0.071	0.071	0.034	-0.001	0.073**
	(0.035)	(0.011)	(0.037)	(0.034)	(0.037)	(0.023)	(0.037)	(0.035)
Age	-0.001***	0.001	-0.001	-0.001**	-0.001***	-0.001	-0.001	0.001
	(0.000)	(0.001)	(0.001)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
Observations	4397	481	756	758	4397	698	424	439
Adjusted R^2	0.021	0.012	0.021	0.015	0.048	0.06	0.022	0.034
Notes:	Robust standard errors in parentheses							
	*** p<0.01, ** p<0.05, * p<0.1							

Tables 5 and 6 present a selection of results for specific indices for brevity, using all four happiness indicators, but the results are quite robust across all mobility indices. For the group of individuals with low mobility it is evident that their outcome is associated with high education and higher income levels. They are most often female and are always married. Being unemployed with the past 26 weeks is seen to be negatively associated with low mobility. In other words, people with low mobility in happiness levels do not experience unemployment in a 6 month period.[5] We also observed that they are most often not likely to experience a divorce, depending upon the happiness variable that is being used. Low mobility individuals also have fewer children.

We estimate a similar logit model for high mobility, given by:

$$high_i = \rho + \mathbf{P}_i \varsigma \tag{18}$$

where $high_i = 1$ if the mobility index for individual i is greater than or equal to 0.65, otherwise $high_i = 0$. $\mathbf{P}_i$ is a vector of socio-economic characteristics (used in the previous regression models). The average marginal effects of the above estimated model are presented in Tables 7 and 8 below.

[5] We have included the incidence of unemployment within 52 weeks, and have not observed any significant associations with low mobility. We have therefore dropped it from the model.

Table 7: Socio-economic determinants of high mobility, selected indices

	General Happiness				Life Satisfaction			
	Gini	Simpson	Arithmetic	Euclidean	Gini	Simpson	Arithmetic	Euclidean
Female	-0.069***	-0.068***	0.027*	0.119***	0.031**	0.025***	-0.022	-0.144***
	(0.015)	(0.015)	(0.016)	(0.016)	(0.016)	(0.005)	(0.016)	(0.054)
Illness	0.003	0.003	0.006	0.055***	0.030***	0.012***	-0.029***	0.094***
	(0.006)	(0.006)	(0.006)	(0.007)	(0.006)	(0.002)	(0.007)	(0.024)
Married	-0.064***	-0.063***	-0.041**	-0.071***	-0.072***	-0.012***	-0.063***	-0.235***
	(0.016)	(0.016)	(0.017)	(0.017)	(0.018)	(0.005)	(0.017)	(0.061)
Was unemployed	-0.016	-0.016	-0.018	0.059	-0.006**	-0.014	0.020	-0.040
	(0.035)	(0.035)	(0.038)	(0.053)	(0.040)	(0.023)	(0.043)	(0.171)
Unemployed for 26weeks	0.009	0.009	0.003	0.041***	0.110***	0.015***	0.043***	0.161***
	(0.047)	(0.047)	(0.012)	(0.016)	(0.056)	(0.004)	(0.016)	(0.062)
Education	-0.010***	-0.010***	0.007	-0.047**	-0.015***	0.002**	0.001	0.005***
	(0.002)	(0.002)	(0.019)	(0.020)	(0.007)	(0.001)	(0.001)	(0.002)
Log Income	-0.028***	-0.028***	-0.001	-0.007	0.041***	0.004*	-0.004*	-0.021**
	(0.006)	(0.006)	(0.001)	(0.001)	(0.008)	(0.002)	(0.002)	(0.009)
Number of children	0.002	0.002	0.003	0.010***	0.030	0.001	-0.022*	-0.079***
	(0.007)	(0.007)	(0.002)	(0.002)	(0.037)	(0.002)	(0.008)	(0.027)
Divorced	-0.041	-0.041	-0.048	-0.041	0.030	0.023	-0.051*	0.005
	(0.033)	(0.033)	(0.033)	(0.037)	(0.037)	(0.032)	(0.029)	(0.029)
Age	-0.001	0.001	-0.001	-0.001	-0.002	0.001	0.001	-0.002
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.000)
Observations	653	569	440	742	1437	2445	1402	1911
Adjusted R^2	0.036	0.045	0.042	0.025	0.064	0.093	0.073	0.066
Notes:	Robust standard errors in parentheses							
	*** p<0.01, ** p<0.05, * p<0.1							

Table 8: Socio-economic determinants of high mobility, selected indices

	Subjective Wellbeing, Likert				Subjective Wellbeing, Caseness			
	Gini	Simpson	Arithmetic	Euclidean	Gini	Simpson	Arithmetic	Euclidean
Female	-0.070***	-0.017***	0.602***	8.774***	0.024***	-0.011	0.342***	3.231***
	(0.015)	(0.003)	(0.072)	(1.122)	(0.004)	(0.016)	(0.050)	(0.406)
Illness	0.002	0.011***	0.324***	4.071***	0.018***	0.051***	0.154***	1.168***
	(0.006)	(0.001)	(0.030)	(0.494)	(0.002)	(0.006)	(0.021)	(0.117)
Married	-0.092***	-0.007***	-0.245***	-3.501***	-0.011***	-0.082***	-0.079	-0.838*
	(0.017)	(0.003)	(0.077)	(1.215)	(0.004)	(0.019)	(0.052)	(0.439)
Was unemployed	-0.025	0.003	0.358	5.582	0.015*	-0.051	0.224	2.356
	(0.037)	(0.011)	(0.265)	(4.472)	(0.009)	(0.041)	(0.158)	(1.465)
Unemployed for 26weeks	-0.002	-0.003	0.257***	4.499***	0.035	0.054	0.102**	0.933**
	(0.051)	(0.002)	(0.069)	(1.168)	(0.065)	(0.056)	(0.044)	(0.374)
Education	-0.012***	0.002***	-0.028***	-0.356***	0.001*	-0.007***	-0.008***	-0.087***
	(0.002)	(0.000)	(0.003)	(0.042)	(0.001)	(0.002)	(0.002)	(0.015)
Log Income	-0.032***	0.002	0.057***	0.777***	0.004*	-0.027***	-0.018**	0.195***
	(0.006)	(0.001)	(0.010)	(0.157)	(0.002)	(0.007)	(0.008)	(0.061)
Number of children	0.004	(0.000)	0.026	0.173	0.005**	0.023***	0.034	0.247
	(0.008)	(0.001)	(0.035)	(0.554)	(0.002)	(0.008)	(0.024)	(0.194)
Divorced	-0.073**	-0.032**	-0.073**	-0.074**	-0.035	0.002	-0.078**	-0.111***
	(0.035)	(0.016)	(0.035)	(0.035)	(0.032)	(0.037)	(0.035)	(0.037)
Age	0.001	0.001	0.001	-0.001	-0.001	-0.001	-0.001	0.001
	(0.000)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)
Observations	2934	2924	2952	2979	2852	2975	2240	2479
Adjusted R^2	0.056	0.167	0.146	0.111	0.081	0.111	0.073	0.056
Notes:	Robust standard errors in parentheses							
	*** p<0.01, ** p<0.05, * p<0.1							

Tables 7 and 8 present regressions with all four mobility indices, using all four happiness indicators, and the results are robust across all mobility indices. The high mobility individual has a very different set of socio-economic characteristics than that of the low mobility individual. The results reveal they are almost always male, with low levels of education and income, and are almost always single and mostly young. In some cases there is significant association with illness. In addition, they are often not divorced. In some cases, having more children is associated with high mobility. Interestingly, none of the unemployment variables are strongly associated with high mobility.

6 Conclusion

In this paper we have conceptualised intra-generational mobility over several periods when variables take values corresponding to either ordered or unordered categories, a topic not discussed in the mobility literature as yet. We propose concepts and related desirable properties of maximum and minimum mobility, along with mobility-inducing transformations. A number of functional forms for indices of individual mobility and social mobility are also proposed. We introduce two measurement frameworks: one suitable for both categorical and ordinal variables, and another one for only ordinal variables. We show that the indices belonging to the first framework measure mobility as diversity or instability in people's status for the period in question; whereas those from the second framework capture a notion of mobility as average distance traveled between adjacent periods. In both cases, our method differs from previous efforts in that we measure mobility explicitly at the individual level (with a later option for aggregations at the social level). Thus we avoid imposing an assumption of population homogeneity, whereby "the same transition rates apply to all individuals in the group" (Shorrocks, 1976, p. 567). This property is pervasive in the traditional literature, either implicitly or explicitly.

Our illustration looked at the degree of mobility in life satisfaction measures in the UK. Using these indices we have identified that individuals who experience low mobility in their life satisfaction are educated and well-off, most often female, married, in good health, and with an insignificant incidence of any unemployment spells. Individuals with high mobility in life satisfaction, on the other hand, are almost always male, single and young, with low levels of education and income, and often ill. The indices were useful in uncovering interesting relationships between some key socio-economic characteristics and the degree of instability of life satisfaction responses in the British population. There are several avenues for future research. The obvious one is to conceptualize and propose measurements for directional mobility (e.g. upward or downward). Another avenue is the development of statistical inference tools for the proposed mobility measures.

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A Appendix: Life satisfaction variables used from BHPS

Here we describe the variables that have been used from the BHPS, with the questionnaire question and the response categories.

General happiness (direct definition from round A): 4 point scale (decreasing levels of happiness with increasing)

GHQ: general happiness

Question: Have you recently been feeling reasonably happy, all things considered?

Responses: More than usual 1, Same as usual 2 , Less so 3, Much less 4

Life satisfaction (direct definition from round H): 7 point scale

LFSATO: Satisfaction with: life overall

Question: How disatisfied or satisfied are you with your life overall

Responses: Not satisfied at all, 1, on an increasing scale to Completely satisfied, 7.

Subjective well-being, Likert (definition from round H): 36 point scale

Subjective wellbeing (GHQ) 1: Likert (derived variable)

Subjective well-being, Caseness (definition from round H), 12 point scale:

Subjective wellbeing (GHQ) 2: Caseness (derived variable)

B Appendix: Summary statistics

Table 9: Summary statistics of key variables in use

Variable	Obs	Mean	Std. Dev.	Min	Max
General Happiness					
Gini	58264	0.220	0.189	0	1
Simpson	58264	0.442	0.280	0	1
Arithmetic	58264	0.364	0.272	0	1.647
Euclidean	58264	0.456	0.431	0	3.529
Life Satisfaction					
Gini	65884	0.229	0.141	0	0.871
Simpson	65884	0.774	0.264	0	1.263
Arithmetic	65884	0.619	0.364	0	3.000
Euclidean	65884	1.035	1.073	0	11.909
Subjective Wellbeing, Likert					
Gini	53770	0.203	0.09	0	0.557
Simpson	53770	0.378	0.046	0	0.432
Arithmetic	53770	0.346	0.193	0	1.329
Euclidean	53770	2.664	2.98	0	26.57
Subjective Wellbeing, Caseness					
Gini	53770	0.134	0.112	0	0.529
Simpson	53770	0.583	0.277	0	0.961
Arithmetic	53770	1.744	1.295	0	7.412
Euclidean	53770	9.254	9.929	0	74.824
Female	84523	0.553	0.497	0	1
Illness	84523	1.348	1.156	0	4
Married	84420	0.620	0.485	0	1
Was Unemployed	84523	0.050	0.217	0	1
Was Unemployed for 26 weeks	84523	0.021	0.143	0	1
Age	84523	46.003	16.738	15	97
Education level	84523	5.442	3.394	0	11
Log of Income	84523	9.049	1.188	-0.732	13.152
Number of children	84523	0.590	0.953	0	8
Divorced	84420	0.056	0.230	0	1

C Appendix: Proofs of propositions

C.1 Proof of proposition 1

Sufficiency:

If $\mathcal{M}_n$ is symmetric and concave, then it is also Schur-concave. Now, by definition of Schur-concavity, if $P_Y = BP_X$, where B is a bi-stochastic matrix, then it must be the case that $\mathcal{M}_Y \geqslant \mathcal{M}_X$, so axiom 4 is fulfilled.

As for axiom 3, let $M'_X(i) \equiv \frac{\partial \mathcal{M}_X}{\partial p_n(i)}$. Then due to symmetry and concavity it must be the case that: $M'_X(i) = M'_X(j)$ if and only if $p_n(i) = p_n(j)$, and $M'_X(i) < M'_X(j)$ if and only if $p_n(i) > p_n(j)$. So, if Y is obtained from X by PPD involving $p_n(i) > p_n(j)$, we can write $\mathcal{M}_Y - \mathcal{M}_X \simeq \delta[M'_X(j) - M'_X(i)] > 0$. Thus, axiom 3 is also fulfilled.

Now since $\mathcal{M}_n$ is a Schur-concave function (due to being both concave and symmetric) then, following Arnold (2007, p. 2), it is easy to show that any index $\mathcal{M}_n$ will reach its maximum value if and only if $p_n(1) = p_n(2) = ... = p_n(S)$. Essentially, we cannot smooth further a distribution characterized by $p_n(1) = p_n(2) = ... = p_n(S)$ with uniform majorization, or PPD specifically. Therefore the index reaches its maximum value when evaluated with that distribution. With appropriate normalization so that such value is 1, then the index fulfills axiom 1.

Likewise, as explained in the proof of proposition 2 below, note that Muirhead's theorem (Marshall et al., 2010, pp. 7-8) implies that any probability Lorenz curve can be obtained from the probability Lorenz curve of a distribution characterized by minimum mobility through a sequence of PPD, because only in the case of a distribution characterized by minimum mobility $L_X(i) = 0 \quad \forall i \in [1, 2, ..., S-1]$. Then, due to concavity, the index cannot take values below that corresponding to a distribution characterized by $\exists i | p_n(i) = 1$ (based on the reasoning applied to prove the sufficiency of concavity for the fulfillment axiom 3). The index's symmetry guarantees that all distributions characterized by $\exists i | p_n(i) = 1$ attain the same value. Concavity across the whole domain of the function also guarantees that the minimum value is unique, so it is attained only when $\exists i | p_n(i) = 1$. With appropriate normalization so that such value is 0, the index fulfills axiom 2.

Necessity:

Necessity of symmetry: Let $P_Y = QP_X$, where Q is a permutation matrix (so that P_Y has the same elements as P_X, but in different order). Then if P_X is characterized by $\exists i | p_n(i) = 1$, it should also be the case that P_Y is a vector with all elements equal to 0, except for one equal to 1. By definition, without symmetry we could have $\mathcal{M}_Y \neq \mathcal{M}_X$ and then $\mathcal{M}_n$ would violate axiom 2.

Necessity of concavity: Given symmetry, if $\mathcal{M}_n$ is not concave, we could have a situation in which Y is obtained from X by PPD involving $p_n(i) > p_n(j)$ and still $\mathcal{M}_Y - \mathcal{M}_X \simeq \delta[M'_X(j) - M'_X(i)] \leqslant 0$. Thus, axiom 3 would be violated. Likewise, we can consider situations in which lack of concavity would lead to violation of axiom 2. For example, a symmetric but not additively separable function like $\Pi_n \equiv \frac{1}{S^S} \prod_{i=1}^{S} p_n(i)$ satisfies axiom 1 but violates axiom 2 because $\Pi_n = 0 \leftrightarrow \exists j | p_n(j) = 0$, which includes the situations of minimum mobility along with more mobile distributions.

C.2 Proof of proposition 2

Note that, naturally: $\sum_{i=1}^{S} p_X(i) = \sum_{i=1}^{S} p_Y(i) = 1$. Therefore, given the definitions in 5, Muirhead's theorem (Marshall et al., 2010, pp. 7-8) applies. The theorem, translated in terms of definition 5, states that $L_X(i) \geqslant L_Y(i) \quad \forall i \in [1,2,...,S]$, $\exists j \in [1,2,...,S] | L_X(j) > L_Y(j)$ and $\sum_{i=1}^{S} p_X(i) = \sum_{i=1}^{S} p_Y(i)$ if and only if X can be obtained from Y through a sequence of PPD transfers. Finally, the same sequence of PPD transfers will ensure that $\mathcal{M}_X > \mathcal{M}_Y$ for any $\mathcal{M}_n$ sensitive to those transfers according to axiom 3.

C.3 Proof of proposition 3

Sufficiency:

Given the symmetry of $\mathcal{B}_n$, it is clear that it satisfies axiom 8. Satisfaction of axiom 7 is also guaranteed by $g' > 0$ and $h' > 0$. Given symmetry, $h' > 0$, $g' > 0$ and additive separability, axiom 5 is also satisfied, as the maximum value (which can be normalized to be equal to 1), i.e. $h[\sum_{t=2}^{T} g(1)]$, is global. Likewise, symmetry, $h' > 0$, $g' > 0$ and additive separability, also guarantee satisfaction of axiom 6, since the minimum value (which can be normalized to be equal to 0) is only attained with $h[\sum_{t=2}^{T} g(0)]$.

Necessity:

If $\mathcal{B}_n$ were not symmetric then axiom 8 would be violated. Likewise if either $h' \leqslant 0$, $g' \leqslant 0$ or both, then axiom 7 would also be violated. Without additive separability in the argument of h, the value of $h[0,0,...,0]$ (i.e. when all distances are zero) could be obtained also with vectors D_n featuring non-zero distance elements. Therefore axiom 2 would be violated. For example, consider a geometric mean $G_n = \prod_{t=2}^{T} d_{nt}^{\frac{1}{T-1}}$. G_n is symmetric and strictly increasing in any d_{nt} as long as $d_{nt} > 0 \ \forall t$. Likewise $G_n = 1$ if and only if $d_{nt} = 1 \ \forall t$. However $G_n = 0$ if and only if $\exists t | d_{nt} = 0$, which includes the case of minimum ordinal mobility but also other cases of higher mobility. Therefore it clearly violates axiom 2. In the case of symmetric functions, we avoid this problem only with additive separability.

C.4 Proof of proposition 4

First note that, given our definition of h it must be the case that $\mathcal{B}_X > \mathcal{B}_Y \leftrightarrow \sum_{t=2}^{T} g(d_{Xt}) > \sum_{t=2}^{T} g(d_{Yt})$. Then we can define:

$$\Delta\mathcal{B} \equiv \mathcal{B}_X - \mathcal{B}_Y = \sum_{i=0}^{1} g(i)\Delta\phi(i) \tag{19}$$

, where $\Delta\phi(i) \equiv \phi_X(i) - \phi_Y(i)$. We can do this because once S is set, d_{nt} can only take a limited set of values, basically $d_{nt} = 0, \frac{1}{S-1}, \frac{2}{S-1}, ..., 1$. Likewise we can define: $\Delta\Phi(i) \equiv \Phi_X(i) - \Phi_Y(i)$. Finally, summing 19 by parts using Abel's formula we get:

$$\Delta\mathcal{B} = -\sum_{i=\frac{1}{S-1}}^{1} [g(i) - g(i - \frac{1}{S-1})]\Delta\Phi(i - \frac{1}{S-1}) \tag{20}$$

Now note that $g(i) - g(i - \frac{1}{S-1}) > 0$ since we are working with $g' > 0$. Then, from 20 it is easy to see that $\Delta\Phi(i - \frac{1}{S-1}) \leqslant 0 \ \forall i \ \wedge \ \exists j \in [\frac{1}{S-1}, \frac{2}{S-1}, ..., 1] | \Delta\Phi(i - \frac{1}{S-1}) < 0$ implies

$\Delta\mathcal{B} > 0$. This proves the sufficiency of the condition on $\Delta\Phi$. However the condition on $\Delta\Phi$ is also necessary to secure $\Delta\mathcal{B} > 0$, because if it were not (e.g. if it were the case that: $\exists j \in [\frac{1}{S-1}, \frac{2}{S-1}, ..., 1] | \Delta\Phi(i - \frac{1}{S-1}) > 0$), then we could find an admissible function g that would yield $\Delta\mathcal{B} < 0$.